Algorithm II: Homework 0

SKE Data Structure and Algorithm 2/2019

Introduction

Grading policy

- **Intuition based, not solution based:** This homework will be graded based on your attempt and not your solution. Giving a completely wrong answer will **not**, and never make your score zero.
- **Think of them as an long-term brain teaser:** Don't complete them in one day, or you'll missed the chances of perfecting your understandings and perspective of the problems little by little.

Ethics and Code of Conduct

- **Zero tolerance on plagiarism:** This course enforces a zero tolerance policy on plagiarism.
- **Wrong answer is better than correct but copied answer:** The scoring criteria is designed to heavily penalise copied answers.
- **No submission is better than turning in copied submission:** The scoring criteria is designed to motivate students to submit their original work.
- **Feel free to discuss:** Discussion among friends and colleagues are completely normal, and we encourage that. However, please **indicate your collaborators**, as it will be used to track works' originality.

Assigned Questions

From your student ID in the format of `XX1054WXYZ`,

- If W is odd then complete question 1.1.
- If W is even then complete question 1.2.
- If X is odd then complete question 2.1.
- If X is even then complete question 2.2.
- If Y is in {1, 4, 7, 0} then complete question 3.1
- If Y is in {2, 5, 8} then complete question 3.2
- If Y is in {3, 6, 9} then complete question 3.3
- If Z is in {1, 4, 7, 0} then complete question 4.1
- If Z is in {2, 5, 8} then complete question 4.2
- If Z is in {3, 6, 9} then complete question 4.3

1 Revisiting proofs 1

These assignments were adapted from ***Discrete Mathematics****, Douglas E. Ensley and J. Winston Crawley, Wiley.*

1.1 Implication A

Prove or disprove that for every integer $n \geq 1$, if n is odd then $n^2 + 4$ is a prime number.

1.2 Implication B

Prove or disprove that for every positive integer n, $2n^3$ is divisible by 3.

2 Revisiting proofs 2

These assignments were adapted from ***Discrete Mathematics****, Douglas E. Ensley and J. Winston Crawley, Wiley.*

2.1 Implication C

Prove or disprove that for every positive integer n, if n is odd then $n^3 - n$ is divisible by 4.

2.2 Implication D

Prove or disprove that for all integers $n > 4$, if n can be expressed in the term of z^2 for any integers z, then $n - 1$ is not a prime number.

2.3 Implication E

Prove or disprove that for all integers n, if n^2 is even then n is even. *Hint: Use contrapositive.*

3 Revisiting proofs 3

Parts of these assignments were adapted from ***Discrete Mathematics****, Douglas E. Ensley and J. Winston Crawley, Wiley.*

3.1 Exhaust A

Proof that if an integer n can be written in the form of z^3 for any integers z, then it must satisfy one of the following critera:

- n must be a multiple of 9,
- n must be 1 more than a multiple of 9, or
- n must be 1 less than a multiple of 9.

3.2 Exhaust B

Prove or disprove that for any integer n, $n^2 + 2$ is even.

3.3 Exhaust C

Prove or disprove that every integer not divisible by 3 had a square that is of the form $3k + 1$.

4 Revisiting proofs 4

In this section, use proof by induction to prove the given statement. Make sure to clearly **indicate your steps of induction** along with the **induction hypothesis (I.H.)**.

4.1 Induction A

Prove that $n^2 + n$ is even for any positive integers n.

4.2 Induction B

Prove that $\sum_{i=1}^{N} i = \frac{(n)(n+1)}{2}$ for any positive integers N.

4.3 Induction C

Prove that $\sum_{i=1}^{N} i = \frac{(n)(n+1)(2n+1)}{6}$ for any positive integers N.